Lessons from the Salvadori Classrooms
LESSON TITLE: Morphing The Room
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TOPIC: School
SSLAM: School / Math / Explore

Morphing the Room

Content Focus: Math

- Measurement: Objects and Units
- Modeling: Quantitative Relationships
- Modeling: Spatial Reasoning

Content Focus: Technology

- Construction Technologies

Content Focus: Built Environment

- Architectural Drawing / 2D
- Measuring/Estimating
- Scale/Proportion

Performance Outcome(s)

- Draw a plan for an addition to increase the area of your classroom by fifty percent using the most efficient rectangle.

Standards/Interdisciplinary Connections

					
S	S	**L**	A	**M**	T
Science	Social Studies	**Language Arts**	Art - Visual	**Math**	Technology

How To Read The Symbols: The symbols in **bold** indicate the subject standards that this lesson satisfies.

Lesson Outline (1 - 3 lessons)

1. Motivation
2. The Challenge
3. Draw A Plan Of The Room
4. Draw An Addition To The Room
5. Morph The Room
6. Reflect
7. Extensions And Variations

Salvadori Prerequisites

- FOUNDATION - BASIC SKILLS: Working with Scale (Activity #3); Measuring/Estimating (Activity #2); Architectural Drawing / 2D (Activity #2)
- Familiarity with area and perimeter

CONCEPTS

- Shapes with the same area can have many different integral perimeters and an infinite number of non-integral perimeters, and shapes with the same perimeter can have many different integral areas and an infinite number of non-integral areas.
- For a given rectangular area, the greater the difference between the values of its length and width, the greater its perimeter.
- The concept of efficiency as it applies to space and materials is an important one in architecture, as it affects designs the design and budget of any architectural project.

RESOURCES

On-Line

- http://hem.dis.anl.gov/eehem/00/000112.html: This is an interesting site explaining the correlation between a building plan's perimeter and energy use. **[need new link]**

MATERIALS

Facilitator

- O: Q&A

Students

- O: Activity Sheet: Area & Perimeter
- Yardsticks or tape measures, rulers, triangles, pencils, graph paper

MOTIVATION

Hand out a blank piece of paper and a ruler to each student.

- *What is perimeter of this piece of paper? How can you find out?* ⭘: Q&A

Give students a few minutes to determine the perimeter of the paper.

On the piece of paper, draw a shape that has the same perimeter as the paper itself. Do not draw along the edges of the paper.

- *Is this possible?*

Students take a few minutes to draw a shape.

- *What were some shapes you came up with?*
- *How many sides does your shape have?*
- *Is there a minimum number of sides that the shape can have and still fit on the paper?* ⭘: Q&A
- *How many possible shapes with the same perimeter are there? (There are an infinite number of shapes of the same perimeter.)*
- *Is it possible to draw a shape that has the same area as the piece of paper on the paper (without drawing along the edges)? Why or why not?* ⭘: Q&A

THE CHALLENGE

Is there anything you would like to add to our classroom? What would we put in here if we had more room? More desks? A computer area? More students? Or would it be nice to have some extra open space?

Imagine that we had the chance to make our classroom bigger. Your challenge is to draw a plan for an addition to increase the area of our classroom by 50%.

DRAW A PLAN OF THE ROOM

- *What does it mean to increase the area of a room by 50%?*
- *What are the steps we will have to take to determine the area of the addition?*

In groups, students use tape measures to determine the dimensions of the room. Alternatively, they can use the estimating techniques described in Basic Skills: Measuring / Estimating to estimate the dimensions of the room.

Students round their measurements to the nearest foot and use them to draw a plan of the room on graph paper, at a scale of 1 square = 1 square foot (for an added challenge they can measure out the plan on blank paper at a scale of 1/8" = 1' or ¼" = 1').

DRAW AN ADDITION TO THE ROOM

- *What is the area of the room?*

Students calculate the area of the room and compare results to ensure that everyone agrees.

- *How much extra space would we need to increase the area of our room by 50%?*
- *If we were able to build an addition to our room, where would be the best place to build it?* ⭘: Q&A
- *What would be the best shape for an addition to our room?*
- *Why are most rooms rectangular?* ⭘: Q&A

Let's assume we want to keep the shape of our room rectangular. On your plans, draw the addition adding 50% more area to the room while keeping the overall shape of the room rectangular. Make sure you add the extra space to the side of the room which we decided was best suited for an addition.

- *How will you determine the dimensions of the addition?* ⭘: Q&A

Students take time to draw their revised floor plan and compare results.

MORPH THE ROOM

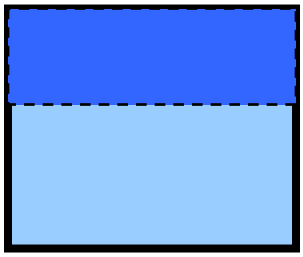
- *What are some things that architects and engineers have to consider when they design an addition?* ⭘: Q&A
- *As you know, building extra space can be very expensive. Let's see if we can design an addition to our classroom that is cheaper to build than the one you have just drawn.*

Students draw or trace another floor plan of the classroom as it exists.

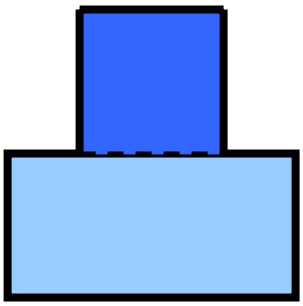
- *In order to build the addition, the builders would have to knock down existing wall, add new walls, and add floor area. The floor area is set – we can't save any money on that because we've already decided that we need an extra 50%. But what about the lengths of wall that have to be knocked down and built up? Is it possible to change the perimeter of the addition without changing the area?*

- *Let's assume that we have permission to change the shape of the room. The overall shape of the room no longer has to be a rectangle, though for practical purposes, the addition must be rectangular (so instead of looking like one rectangle the room can have the shape of two adjoining rectangles).*
- *Remember, we want to minimize the amount of wall area that has to be knocked down and built up. Since the height of the room is also set, the only factor we can change is the total length of the walls. This means that we want to minimize the perimeter of the addition.*

A simple classroom plan.



A plan showing an addition of 50% of the original area in which the overall rectangular shape has been maintained.



A plan showing an addition of 50% of the original area in which the overall rectangular shape has not been maintained.

- *What is the perimeter of the addition that you drew onto your original plan?*
- *Now start experimenting with different rectangular additions. Your only restrictions are that the area must remain the same and that the side of the addition adjacent to the existing classroom cannot extend past it (for example, if the wall of the classroom to which you wish to add the extension is 30 feet long, that side of the addition cannot be more than 30 feet long).*

Distribute the Activity Sheet. ⊙: Activity Sheet: Area and Perimeter

Students explore the problem using the chart on the ⊙: Activity Sheet.

Students compare results.

- *Do you notice any correlation between the length of the perimeter and the difference between the length and width?* ⊙: Q&A
- *What type of rectangle is the most efficient?* ⊙: Q&A
- *How many rectangles of different dimensions can share the same area of the addition?* ⊙: Q&A

REFLECTION

- *If you were to actually design an addition to your classroom, would you use the most efficient rectangle in your plan or would you use your original design, in which the entire room forms a rectangle? What are the advantages and disadvantages of each design? Does a square room "feel" very different than a long narrow room with walls of equal height?*
- *What is the difference between the length and the width of the most efficient rectangle?*
- *Why do you think the square is the most efficient of all shapes (as defined by the ratio of its perimeter to its area)?*
- *How well did you meet your challenge? What would you do differently next time?*

EXTENSIONS AND VARIATIONS

- Students explore the relationship between the perimeters and areas of non-rectangular shapes. *What did you find? Which are the most efficient?*
- Reverse the exercise. *For a given perimeter, what is the greatest possible rectangular area can you have?* (The result, $A = (P/4)^2$, is often surprising to students.)
- Students develop their modified floor plans to show the placement of the classroom furniture. *Can you use the old furniture or do you need new furniture? Why?*
- Students graph the equation $A = lw$. (Help them to see that the solution is a hyperbola representing an infinite number of solutions to this equation and thus an infinite number of solutions to the two dimensions of the rectangle whose area is given.)
- Students graph the linear equation $P = 2l + 2w$ where P equals the perimeter of your classroom. (You can then graph the two equations on the same coordinate plane and discuss what you can learn from this superposition with the class.)
- Add volume to the equation. What happens to the volume of the room if its height and floor area remain unchanged while its perimeter changes?

MIDDLE SCHOOL STANDARDS

Language Arts

- Speaking, Listening, and Viewing (E3a, b, c)

Math

- Number and Operations
- Geometry
- Measurement
- Problem Solving
- Communication
- Representation

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MOTIVATION

- What is perimeter of this piece of paper? How can you find out?

  The perimeter of a piece of 8 ½" x 11" paper = 8 ½"+8 ½"+11"+11"= 39".

- Is there a minimum number of sides that the shape can have and still fit on the paper?

  A curvilinear shape of the same perimeter could have one "side" and still fit on the piece of paper. If the shape has no curved sides, then it must have a minimum of five sides in order to fit on the piece of paper, since no four-sided shape other than the exact shape of the paper itself could fit within the confines of the paper's edges and still have the same perimeter.

- Is it possible to draw a shape that has the same area as the piece of paper on the paper (without drawing along the edges)? Why or why not?

  It is not possible to draw a shape with the same area on the paper without drawing along the edges of the paper because any shape that does not include every part of the paper will necessarily have a smaller area.

DRAW AN ADDITION TO THE ROOM

- If we were able to build an addition to our room, where would be the best place to build it?

  There are many factors that go into decided how to best expand an existing space. In the case of the classroom, the most obvious choice would be to build an addition on the exterior of the school building, along the wall facing outward. Expansions along any other wall would just be taking space away from another part of the school building and would therefore not add any net space and potentially disrupt the function of the surrounding areas in the building. If your classroom has no wall separating it from the outside, decide where the expansion could be built so as to cause the minimum amount of disruption. Expanding into an adjacent classroom, for example, would probably cause less disruption than expanding into the hallway.

- Why are most rooms rectangular?

  Most rooms are rectangular because rectangles are relatively efficient shapes when it comes to enclosing space (as far as their perimeter to area ratio is concerned). While circles are more efficient in this regard, circles do not fit together well. In general, rectangles are the most common shapes for rooms because right angles are the most practical in terms of arranging rooms together and in arranging space and furniture within a room (imagine trying to fit desks and bookcases against the walls of a circular room or in the corners of a triangular room).

- How will you determine the dimensions of the addition?

Morphing The Room

Since students, in this exercise, are maintaining a rectangular shape for the room, if they know which wall they are adding the addition to they already have the length of one side of the addition. With the length of one side and the area determined, the other dimensions of the rectangle can be easily calculated.

MORPH THE ROOM

- What are some things that architects and engineer have to consider when they design an addition?

  There are many things that architects must consider when they design an addition. These include aesthetic factors, such as how the addition will look on both the interior and the exterior of the building, practical considerations, such as how the addition will be supported structurally and how it will affect the amount of light that enters a building, and financial considerations, such as material and labor costs and the extra costs of heating and lighting more space.

- Do you notice any correlation between the length of the perimeter and the difference between the length and width?

  The greater the difference between the length and the width of the room, the greater its perimeter is.

- What type of rectangle is the most efficient?

  The square is the most efficient rectangle, in that for any given area it has the smallest perimeter.

- How many rectangles of different dimensions can share the same area of the addition?

  There are an infinite number of rectangles that can share a given area, even with a limit on the size of one of the sides. There are, however, a limited number of rectangles with integral side lengths that can share a given area. For example, for an area of 10 square feet, there are only two possible rectangles with integral side lengths: 10'x1' and 2'x5' (or four rectangles if you were to count the different orientations: 10'x1', 5'x2', 2'x5' and 1'x10').

name(s) date

Morphing The Room

Area and Perimeter

Activity Sheet

How can we build an addition to the classroom while minimizing the amount of wall space we need to knock down and build up?

Another way to put the question is: What is most ***efficient*** rectangle?

In this case, efficiency means encompassing a given area while having the smallest possible perimeter.

- First, calculate the perimeter of the addition that you drew in the original drawing (remember, measure just the perimeter of the *addition*, not the entire classroom).
- Now enter the information into the first row of the chart below.
- Fill out the area column of the chart (enter the same number in every box).

Now experiment by using different numbers for the length and width and calculating the results. Remember that the length for two of the sides of the rectangle is limited by the dimensions of the room.

Area (constant)	Length	Width	Perimeter	Difference between length and width

Do you notice any correlation between the length of the perimeter and the difference between the length and width?

What type of rectangle is the most efficient?